

Elements of Mathematics Foundations

EMF Transcript Course Descriptions

EMF01: Operational Systems

This course introduces students to modular arithmetic; operational systems and their properties (commutativity, associativity, neutral elements, invertibility); several non-numeric operations; least common multiple and greatest common divisor; and the geometric concepts of midpoint and reflection.

EMF02: The Integers

This course introduces students to integers; arithmetic operations on integers using the number line and number plane; additive inverses; inequalities; integer-based operational systems and their properties, including the distributive property of multiplication over addition; and two-fold operational systems.

EMF03: Sets, Subsets and Set Operations

This course introduces students to sets and membership in a set; roster names; the empty set; singleton sets; Venn diagrams; subsets and power sets; set operations including intersection, union, set difference and complement; counting the k -element subsets of an n -element set; and the Pascal Formula.

EMF04: Ordered n -Tuples

This course introduces students to ordered pairs and other ordered collections of objects; Cartesian products; taxicab geometry; equations in one and two variables; characteristics of the geometric plane; graphing the solution set of an open sentence; componentwise operations; and operations on subsets.

EMF05: Mappings

This course introduces students to mappings to and onto sets; one-to-one mappings; counting mappings; permutations; composite mappings; and applications with magnifications, fractions and percentages.

EMF06: The Rationals

This course introduces students to the rational numbers and their properties; operations on rational numbers; solving equations of rational numbers through mappings; the absolute value function; and ratio and proportion.

EMF07: The Decimals

This course introduces decimal numbers and arithmetic operations on decimals; decimal approximations of rational numbers; positional notation for decimals; percentages; statistical measures of central tendency; and statistical measures of variation.

EMF08: Probability

This course introduces students to elementary probability theory including one-stage, two-stage, and multistage random experiments; the Product Rule; counting subsets; combinatorics; random digit generators; simulations; and the Monte Carlo method.

EMF09: Number Theory

This course introduces students to elementary number theory including prime and composite numbers; the Twin Prime Conjecture; Goldbach's Conjecture; the Sieve of Eratosthenes; prime factorization; the Transitivity Theorem; relatively prime numbers; Euclid's Lemma; the Division Theorem; number bases and radial expansions; Bezout's Lemma; the Reduction Theorem; mathematical induction; the Fundamental Theorem of Arithmetic; the divisor sum function, σ ; Euler's totient function, ϕ ; and factors in various operational systems.

EMF10: Algebra: Groups, Rings and Fields

This course introduces students to abstract algebra and three of its fundamental structures--groups, rings, and fields. Topics include groups (both Abelian and non-Abelian); the cancellation property; uniqueness of inverses; opposite operations; rings and pseudo-rings; iterated products and sums; the characteristic of a ring; zero divisors; properties of rings and fields (known in high school algebra as the laws or rules of algebra); and solving equations in groups, rings, and fields.

EMF11: Algebra: Relations, Mappings, Equations

This course continues the study of abstract algebra with topics that include relations over fields; set operations with relations; converse of a relation; properties of relations; equivalence relations; mappings as relations; converse of a function; composition of functions; invertible mappings; additive and multiplicative inverse mappings; the power mappings; and cancellation in the solution of equations.

EMF12: Algebra: Relational and Ordered Operational Systems

This course completes the study of abstract algebra with topics that include relational systems; linear arrangements; the transitive property and the trichotomy property; linear orders and linearly ordered groups, rings, and fields; natural orders; dense orders; the Cantorian order on the rational numbers; additive monotonicity; the Extended Monotonicity Property and Theorem; the Extreme Elements Theorem; the Order Theorem; multiplicative monotonicity; the Integer Separation Theorem and one of its important consequences; properties of ordered groups, rings, and fields; and solving inequalities in ordered groups, rings, and fields.

EMF13: Real Functions I

This course begins the study of functions on the real numbers and of trigonometry and continues the study of statistics with topics that include extending the ordered field of rational numbers; nests of decimal intervals; periodic names for rational numbers; coordinatizing the real number plane; constant functions; functions whose graphs are lines and some whose graphs are not lines; the ring of real functions; systems of first degree equations and inequalities; quadratic equations and inequalities; solving quadratic equations graphically and by factorization; completing the square; the standard form and vertex form of a quadratic function; graphing a quadratic function in vertex form by transforming the graph of x -squared geometrically; manipulating square roots; the quadratic formula; the normal distribution; properties of variance; confidence intervals; margin of error; trigonometric ratios and their

relationships; complementary and supplementary angles; the Law of Sines; the Law of Cosines; and degrees vs. radians.

EMF14: Real Functions II

This course completes the study of functions on the real numbers with topics that include sequences and series; factoring real polynomial functions; integer and rational power functions; the Binomial Theorem; the converse of a real function; root functions; the absolute value function; the greatest integer function; growth functions; exponential and logarithmic functions; rational functions; asymptotes; lines of best fit; statistical modeling; extending the real field to the complex field; and polynomial functions over the complex field.

EMF15: Geometry: Incidence and Transformations

This course begins the formal study of Euclidean geometry with topics that include an investigation of geometric objects such as space, planes, lines, and points; incidence properties of points, lines, and planes; common three-, two-, and one-dimensional figures; measurement of distance and angle; properties of betweenness, collinearity, and circular orientation; classification of angles; point-set topology; compass-and-straightedge constructions; parallel and perpendicular lines and planes; transformations such as reflections, translations, rotations, and magnifications; and composition of transformations.

EMF16: Geometry: Congruence and Similarity

This course continues the formal study of Euclidean geometry with topics that include congruent geometric figures, including the SSS, SAS, and ASA rules for triangles; odd and even isometries and their fixed points; subgroups of isometries; types of triangles and their properties; symmetries of triangles, convex quadrilaterals, convex polygons, and circles; magnifications and similitudes of a plane; ratio and proportion; circle geometry; congruence and similarity of arcs; and extensions of relevant findings to three-dimensional space.

EMF17: Geometry: Coordinates and Measurement

This course concludes the formal study of Euclidean geometry with topics that include area of two-dimensional figures; volume and surface area of three-dimensional figures; right triangles; trigonometric identities; Euclidean and taxi distances in a plane; geometry in the context of the real number coordinate plane; vector spaces; isometries in terms of coordinates; geometric representations of algebraically specified sets, including first-degree equations, and the transformations thereof; conic sections; and extensions of relevant findings to space.

EMF18: Precalculus Coda

The course completes the study of Precalculus by bringing together various Precalculus topics covered in earlier EMF courses and adding to that vectors; linear transformations of the real plane; matrices and their use in solving systems of equations; orthogonal mappings; polar coordinates; parametric equations; and limits and continuity.